

15-112 Fundamentals of Programming

Life Lesson

Loosen up. Relax.
Except for rare life-and-death matters, nothing is as important as it first seems

Announcements

- Midterm on Thursday – October 3.
- Assignment 5 Due Tuesday October 8

What are we doing today?

- Recursion

Recursion

- ❑ A method of simplification that involves dividing a problem into simpler subproblems of the same type.
- ❑ A function that calls itself is called a recursive functions

Factorial Example

```
def factorial(n):  
    if n == 0:  
        return 1  
    return n * factorial(n-1)
```

Base Case

Recursive Case

General Form of Recursive Functions

Function (input)

base case – this will stop chain of calls

recursive case – call Function with simpler input

Definition of Factorial

$$F(0) = 1$$

$$F(n) = n * F(n-1)$$

Definition of power

$$P(b,0) = 1$$

$$P(b,e) = b * P(b,e-1)$$

Recursive Power

```
def power(base, exp):  
    if (exp == 0):  
        return 1  
    else:  
        return base * power(base, exp-1)
```

Sum of All elements in a list

$$F([\]) = 0$$

$$F([x, \dots]) = x + F([\dots])$$

Sum of All elements in a list

```
def listSum(list):  
    if (len(list) == 0):  
        return 0  
    else:  
        return list[0] + listSum(list[1:])
```

Exercise

- We want to write a function that counts how many odd numbers exist in a list
- Write the function definition
- Write the recursive function

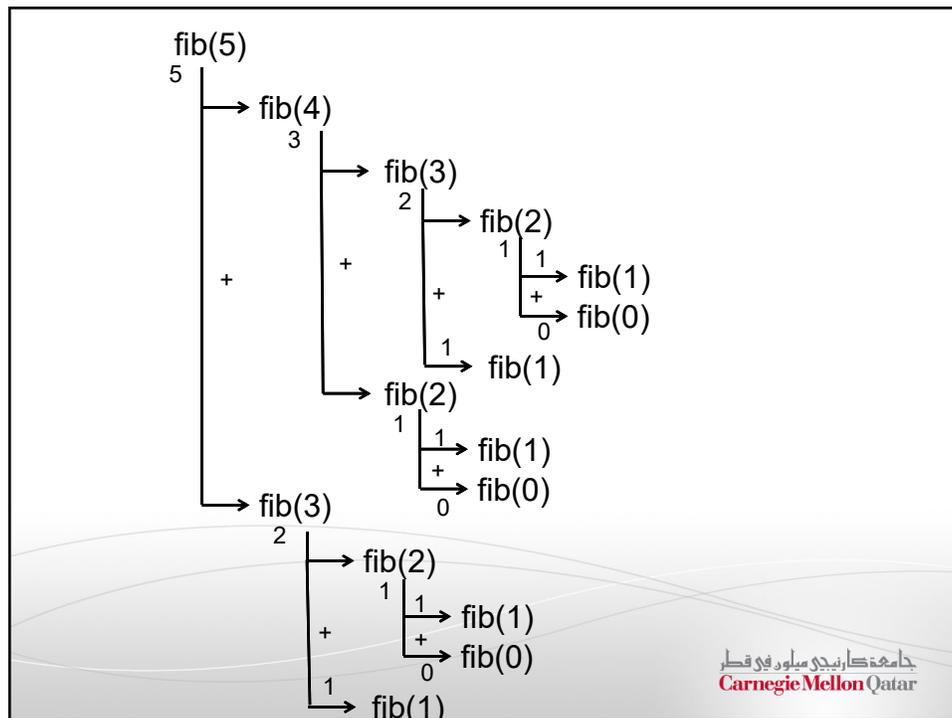
Fibonacci

- Following is the Fibonacci series
0, 1, 1, 2, 3, 5, 8, 13, 21, 34
- In general
$$F_n = F_{n-1} + F_{n-2}$$

Recursive Function

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

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Palindromes

- A palindrome is a word or sentence that reads the same both forwards and backwards
 - rotor
 - madam
 - rats live on no evil star

Recursive palindromes

- How would describe a palindrome recursively?

$F(n) =$ true if $n == ""$
 true if $\text{len}(n) == 1$
 $F(n[1 \rightarrow \text{len}-1])$ if $F[0] == F[-1]$
 false if $F[0] != F[-1]$

Recursive palindrome

```
def palindrome(a):
    if a == "":
        return True
    elif len(a) == 1:
        return True
    elif a[0] == a[len(a)-1]:
        return palindrome(a[1:len(a)-1])
    else:
        return False
```

Do Parenthesis Match?

- “3*(8-4) / (4 – 9)”
- “3*(((8*2)-4) / (4 – 9))”
- “3*(((8*(2+9))-4)(/ ((11*4) – 9)))”

Towers of Hanoi

```
def move(n, frm, to, via):
    if (n == 1):
        print (frm, to)
    else:
        move(n-1, frm, via, to)
        move( 1, frm, to, via)
        move(n-1, via, to, frm)
```

Another Exercise

We have bunnies standing in a line, numbered 1, 2, ... The odd bunnies (1, 3, ..) have the normal 2 ears. The even bunnies (2, 4, ..) we'll say have 3 ears, because they each have a raised foot. Recursively return the number of "ears" in the bunny line,

```
bunnyEars2(0) → 0
bunnyEars2(1) → 2
bunnyEars2(2) → 5
```